Math 1A: Calculus **Instructor: Alexander Paulin**

Handout: Implicit Differentiation

Discussions 201, 203 // 2018-10-10

Curvature

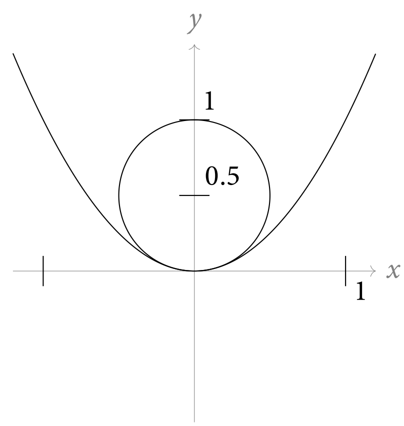
Problem A1. The solution set of the equation

$$x^2 + (y+2)^2 = 4$$

is a circle of radius 2 centered at the point $(0, -2)$. Suppose (x_0, y_0) is a point on the circle. Use implicit differentiation to find the value of $\frac{dy}{dx}$ at that point, in terms of x_0 and y_0.

Problem A2. Consider the function $f(x) = x^2$. Show that the circle from the preceding problem is tangent to the graph of f at the point $(0, 0)$.

Problem A3. Convince yourself that there are in fact *infinitely* many circles which are tangent to the graph of f at the point $(0, 0)$. However, there is a particular circle which, in addition to being tangent to f at $(0, 0)$, is "perfectly nested" in the graph of f.



The equation of this circle is

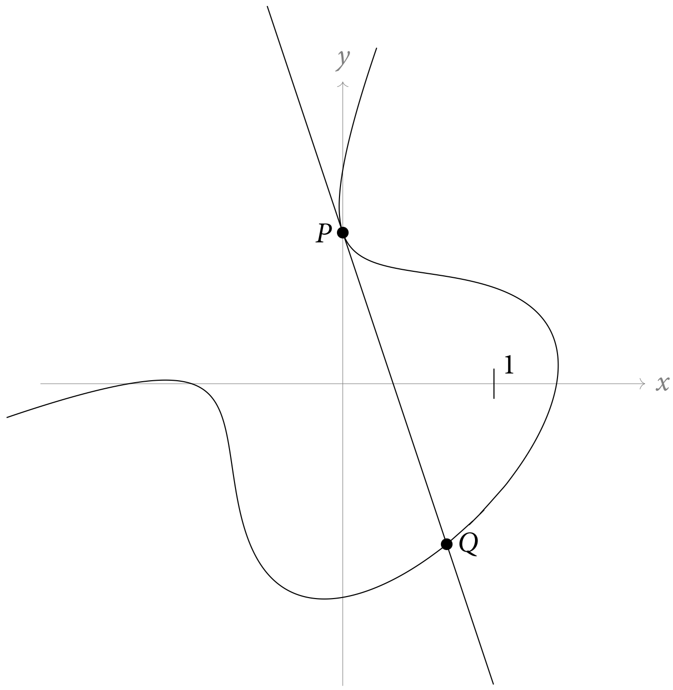
$$x^2 + (y - 1/2)^2 = 1/4.$$

Use implicit differentiation to compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point $(0, 0)$ on the circle. Do these calculations help explain why the circle is so nicely "nested" in the parabola?

A cubic curve

In this part of the worksheet, we will consider the curve given by the equation

$$y^3 - y^2 - 2y - x^3 - x^2 + 2x + 3x^2y - 2xy - 3xy^2 + 2 = 0.$$



Problem B1. Check that the point $P = (0, 1)$ is on the curve.

Problem B2. Write the equation of the tangent line to the curve at the point P.

Problem B3. The tangent line at P intersects the curve at another point Q. Find the coordinates of this other point Q.

If you examine the picture, you'll see that there's another tangent line to the curve which passes through the point Q. (The equation for that one is not so nice.)

Problem (Food for thought)**.** Suppose you were given the equation of a smooth *quartic* (degree 4) curve, as well as a point (x, y) on the curve. Let L be the line through (x, y) that is tangent to the curve. How could you determine the other point(s) (if any) at which L intersects the quartic curve?